An Analysis of Portfolio VaR: Variance-Covariance Approach

Poornima B G* and Y V Reddy**

With the growing exposure and linkages of Indian financial markets with the international financial markets, a rational investor (individual or institutional) would opt to reap the benefits of international investment opportunities by constructing a portfolio which would generate good returns with least risk. At the same time, the investor is unaware of the expected degree of return and risk inherent in the portfolio. This requires predicting the market risk of a portfolio using appropriate model. As such, the study attempts to calculate the portfolio market risk of domestic and international hypothetical portfolio using VaR-CoVaR (Variance-Covariance) model. The daily closing prices for a period ranging from 2000 to 2014 of Nifty Spot (NSR), Nifty Future (NFR), INRUSD currency pair Spot (USR) and INRUSD currency pair Future (UFR) are considered for building hypothetical domestic portfolio. The daily closing prices of BRICS nations, US and UK equity market indices from January 2000 to December 2014 have been considered for international portfolio. The investors are classified as risk-averse, risk-neutral and risk-takers. The study concludes that VaR-CoVaR model provides accurate results at 95% and 90% confidence intervals.

Introduction

When we look from an investor viewpoint, he/she may not be willing to invest in individual assets. This may be due to the fact that the investor is unaware of the expected degree of return and risk borne by individual assets. Hence, the investor chooses to make investments in a portfolio. A portfolio can take any kind of the following combinations—only equities, only bonds, only forex, only commodities, equities and bonds, equities, bonds and forex, equities, bonds, forex and commodities, etc. A portfolio can also be of only domestic investments, only international investments or a combination of domestic and international investments. As the present study focuses only on financial markets, commodity market investment as a part of portfolio is not accounted for. The choice of any of the above portfolio is also based on the degree of risk a rational investor is ready to bear.

BRICS: An Investment Opportunity

BRICS economies gained tremendous attention in recent years. The name BRIC, an acronym, was coined by Goldman Sachs in the year 2001 for Brazil, Russia, India and China. In 2010, South Africa joined BRIC nations and the group was renamed as BRICS. These emerging markets have shown remarkable economic growth that rendered high return for investors. BRICS economies account for 40% of the world population and 25% of the world's GDP. As

* Assistant Professor, Commerce Department, Goa University, Goa 403206, India. E-mail: poornima.bg@unigoa.ac.in

** Professor and Registrar, Goa University, Goa 403206, India; and is the corresponding author. E-mail: yvreddy@unigoa.ac.in

such the international investment trends are shifting from developed markets to emerging economies (Bohn and Tesar, 1996). There has been empirical research which analyzes the economic prospects of BRICS countries and the investment opportunities (Wilson and Purushothaman, 2003). However, one cannot deny the risks inherent in the emerging markets on account of transparency in reporting, regulatory system and compliance challenges. Further, government/bureaucratic intervention are hurdles for the smooth functioning of the markets. The present study also considers the BRICS economies for calculating market risk using equity indices along with the developed markets' (US and UK) equity indices.

This paper makes an attempt to calculate the risk of two hypothetical portfolios—domestic and international—using variance-covariance Value at Risk (VaR) model.

Literature Review

Campbell *et al.* (2001) construct a portfolio selection model which helps the risk manager in allocating financial assets expecting maximum return subject to the constraint of maximum loss set by VaR limits. The authors develop Sharpe Index similar to mean-variance approach. The results show that the model proposed by the authors and mean-variance approach generate identical results when returns are normally distributed. The authors employ two risky assets, namely, US stocks and bonds in their study. The results highlight the influence of both non-normal characteristics of the expected return distribution and the length of investment time horizon on the optimal portfolio selection.

Fusai and Luciano (2001) show that usual VaR measures underestimate portfolio losses, even if the underlying returns are normal. They also study the dependence of the misestimate on the VaR horizon, the initial portfolio mix and the risk aversion of the portfolio manager. The backtesting of conditional VaR confirms the inappropriateness of the usual VaR.

Sentana (2001) provides a unifying approach to mean-variance analysis and VaR. The author explains how fund managers can take investment decisions within the well-known mean-variance allocation framework that satisfies the VaR restrictions imposed on them by regulators by introducing a new type of line to the usual mean-standard deviation diagram, called IsoVaR, which represents all the portfolios that share the same VaR for a fixed probability level.

Ramazan *et al.* (2003) assess the performance of VaR models for daily closing prices of Istanbul Stock Exchange (ISE-100) Index for the period 1987 to 2001 with a total of 3,383 observations. They consider variance-covariance approach, historical simulation, GARCH(1,1), GARCH(1,1)-t, adaptive GPD and nonadaptive GPD models. VaR is calculated for different window sizes of 500, 1000 and 2000 days. The result concludes that GARCH(1,1) and GARCH(1,1)-t models provide high volatile forecasts. Variance-covariance approach, historical simulation, adaptive GPD and nonadaptive GPD models provide stable forecasts.

Alexander and Baptista (2004) compare the VaR and Conditional VaR (CVaR) constraint on mean-variance model for portfolio selection. They show that for a given confidence interval, a CVaR constraint is better than a VaR constraint. Further, a CVaR constraint is more effective than a VaR constraint as a tool to control slightly risk-averse agents, but in the absence of a risk-free security, has a perverse effect in that it is more likely to force highly risk-averse agents to select portfolios with larger standard deviations. However, when the CVaR bound is appropriately larger than the VaR bound or when a risk-free security is present, a CVaR constraint 'dominates' a VaR constraint as a risk management tool.

Chu *et al.* (2006) use returns of the daily closing prices of six international indices. The authors construct three hypothetical portfolios as follows: Portfolio (1): S&P500, FTSE 100 and DAX; Portfolio (2): TAIEX, Nikkei 225 and Hang Seng; Portfolio (3): S&P500, FTSE 100, DAX, TAIEX, Nikkei 225 and Hang Seng. The data was collected from 1990 to 2004. The Power Exponentially Weighted Moving Average (EWMA) method suggested by Guermat and Harris (2002) in conjunction with historical simulation is used for estimating portfolio VaR. The Power EWMA was able to capture volatilities and time varying tail-fatness of financial returns. The backtesting results of Kupiec (1995) suggest that Power EWMA model enhances the estimation accuracy of portfolio VaR.

Alexander and Baptista (2008) evaluate the imposition of VaR constraint using the model suggested by Roll (1992). The authors conclude that constraint mitigates the problem that when an active manager seeks to outperform a benchmark using the mean-Tracking Error Variance (TEV) model, he or she selects an inefficient portfolio.

Hsin (2008) aims to analyze the VaR efficient frontier portfolio selection using polynominal goal programming. The data for the study consists of monthly rates of 10 equity indices of Pacific Rim countries. They are US, Canada, Mexico, Chile, Peru, Japan, Korea, Singapore and Hong Kong. The period studied is for 1991 to 2006. The results demonstrate that polynomial goal programming model is superior to the traditional techniques as it has the ability to consider the risk-return trade-off between the expected return and VaR; as such, the author advises the usage of this model for fund managers and investors.

Gordon *et al.* (2011) demonstrate how the correlation between equity and foreign exchange components in a portfolio can help reduce foreign exchange exposure risk using variance-covariance VaR for a portfolio. The equity indices of Argentina, Brazil, China, India, Mexico, Russia and US are used to decompose portfolio risk. The authors suggest that the investors should consider the correlation between foreign exchange market and equity market while making investments in emerging market equity portfolio.

Marcos and Pablo (2011) use Stochastic Volatility Factor Models (SVFM) to measure and analyze the risk and problems of the portfolio. To what extent the VaR and expected shortfall are sensitive are analyzed by changing the parameters in the model. Linear portfolio positions of assets are compared using SVFM and Black and Scholes Model. Three stocks of the Asian market are considered for the application of the above-mentioned models. The empirical results show that stochastic volatility parameters are significant. The analysis shows that the parameters of the stochastic volatility part are statistically significant and that such parameters make a difference on the risk-return trade-off of the portfolio as well as the dynamics of the risk measures considered.

Carlo *et al.* (2012) consider two portfolios: A and B. Portfolio A is more diversified and it consists of two US stocks each from four different industries. Portfolio B is strongly correlated and it consists of eight US stocks from a single industry. Portfolio A is more realistic from the investor point of view with wide diversified stocks, and Portfolio B is considered as a stress test portfolio. The data covers the period from 1991 to 2008. The study considers multivariate stable-like risk factors, multivariate *t*-like risk factors and multivariate meta-like risk factors and VaR by simulation for forecasting VaR. Backtesting is done for the above models, and it is concluded that meta-*t* and meta-stable laws offer good performances on least diversified portfolios.

Joel *et al.* (2012) consider the daily returns of 48 industry portfolios from 1963 to 2007. The authors have proposed a new risk measure called Partitioned VaR (PVaR) for portfolio optimization by separating the distributions of the returns of the assets as upside risk and downside risk half-spaces. They compare PVaR with the traditional Markowitz mean-variance approach. The results show that PVaR estimates are more useful for portfolio allocations when asset return distributions are asymmetrical.

Brandtner (2013) compares CVaR and mean-variance analysis for portfolio selection. The author does not consider investor's mean-spectral risk preference for the optimal portfolio choice. Rather they model these preferences in the form of spectral utility function which assumes that diversification is never optimal if there is a risk-free asset.

Jang and Park (2016) have used VaR approach for building an optimal portfolio to balance between risky and riskless assets. The author opines that when a fund manager controls asset composition, reactions differ with respect to an increase in only risk aversion and only ambiguity aversion. When the sum of coefficients of risk aversion and ambiguity aversion is fixed, the effect of risk aversion on risky investment dominates the effect of ambiguity aversion in that stock holdings are dramatically smaller in the absence of ambiguity aversion than in its presence.

Theoretical Background

Portfolio Theory and VaR

Portfolio theory has formed the basis on which VaR models are built to measure various risks faced by financial institutions or individual investors. Portfolio theory assumes that a rational investor selects a portfolio considering two aspects, namely, return and risk. As per portfolio theory, an efficient portfolio is said to yield high return for a given risk or a low risk for a given return. In a portfolio theory, what is more important "is the extent to which an individual asset contributes to the overall portfolio risk. This depends on the degree of correlation or covariance of singe asset returns with the returns of other assets in a portfolio" (Dowd, 2002). If the degree of correlation is high, asset's contribution to the overall portfolio risk will be high. Further, in case the correlation is positive, the asset will fail in offsetting the risk and portfolio standard deviation tends to be higher. As such, portfolio theory is very useful for financial analysts in analyzing the extent of interaction of various risks in a portfolio.

Mere standard deviation does not help in assessing the risk of a portfolio. A robust risk measure is given by VaR models. "VaR or the maximum expected loss for a given time horizon is calculated on the basis of the standard portfolio theory which in turn is estimated using standard deviation and correlations between the returns of the various instruments traded" (Dowd, 2002). However, there exists a difference between portfolio theory and VaR. The difference is summarized in Table 1.

Table 1: Difference Between Portfolio Theory and Value at Risk

S. No.	Portfolio Theory	Value at Risk
1.	Risk is interpreted as the standard deviation of the return.	Risk is interpreted as the maximum likely loss.
2.	It presupposes that the returns are normally distributed.	No such assumption is necessary. It can accommodate any kind of distribution.
3.	It is limited for calculating only market risk.	Various VaR models can be used for calculating any type of financial risk.
4.		Variance-covariance VaR is theoretically based on Portfolio Theory.

Source: Dowd (2002), p. 11

Due to the introduction of manifold financial instruments like options, futures and swaps, various types of structured loans like Collateralized Mortgage Obligations (CMO) and Collateralized Debt Obligations (CDO), the portfolios built by the institutional investors with these financial instruments have increased the complexities in assessing the market risks. It has become a challenge for the institutional investors to measure the aggregate risk in a meaningful manner (Angela, 2009). Hence the need to measure the risk in a standardized way which has become possible through VaR models.

Variance-Covariance VaR

JP Morgan's 'RiskMetrics' system developed the Variance-Covariance (VCV) approach. This model is an extension of Markowitz concept of portfolio risk (Jorion, 2001). The model considers the moving average concept in calculating the VaR estimates at a certain confidence interval. This model assumes that the financial-asset returns are normally distributed and hence follow Gaussian probability density function. Hence the returns are described by mean, standard deviation or the variance and the correlation between various market returns.

Correlation between various market returns is given by variance-covariance matrix. "The VCV method assumes that correlation between risk factor remains same" (Learning Curve, 2003). Correlation measures the degree to which the variables are related. For a given portfolio, risk can be reduced provided the assets are positively correlated due to diversification. Diversification in portfolio helps in reducing the total risk which will be less than the sum of the individual assets' risks.

The assumption of normal distribution makes VCV approach estimate VaR easily as skewness and kurtosis are not accounted for. This assumption is a serious limitation of the model as the empirical financial-asset returns exhibit excess kurtosis.

Data and Methodology

In order to calculate the portfolio market risk, the study used variance-covariance VaR model. The daily closing prices of Nifty Spot (NSR), Nifty Future (NFR), INRUSD currency pair Spot (USR) and INRUSD currency pair Future (UFR) are considered for building hypothetical domestic portfolio. The data for NSR and NFR ranges from January 2000 to December 2014.

The data for USR and UFR ranges from August 2008 to December 2014. The daily closing prices of BRICS nations, US and UK market indices from January 2000 to December 2014 have been taken from Bloomberg database. Table 2 gives the list of the nations and the respective equity market indices.

Table 2: List of Countries and Stock Market Indices

S. No.	Country	Equity Market
1.	Brazil	IBOV INDEX
2.	Russia	INDEXCF INDEX
3.	India	NIFTY INDEX
4.	China	SHCOMP INDEX
5.	South Africa	JALSH INDEX
6.	US	NYA INDEX
7.	US	SPX INDEX
8.	UK	UKX INDEX

Two hypothetical portfolios are built: one for domestic investments and the other for international investments. The hypothetical portfolio for domestic investment consists of NFR, NSR, USR and UFR. The hypothetical portfolio for international investments comprises equity indices of five developing nations and two developed nations. Brazil, Russia, India, China and South African equity indices represent the developing countries. US and UK equity markets are considered for developed nations. Further, the investors are placed into three classifications: risk-averse investors, risk-neutral investors and risk-taker investors.

It is assumed that risk-averse investors invest less in the assets with less standard deviation and more with high mean values. The risk-neutral investors invest equally their investments in all the assets of a portfolio. The risk-taker investors invest high amount in the assets with high standard deviation bearing high risks. Accordingly, the study in order to follow the above categorization ranks the standard deviation. First, the standard deviation is calculated and ranks are assigned based on the types of the investors.

Results and Discussion

Descriptive Statistics

The empirical analysis starts with descriptive statistics. Table 3 shows the descriptive statistics of NFR, NSR, UFR and USR daily log normal returns of closing prices which forms the part

Table 3: Descriptive Statistics of Indian Financial Markets				
	NFR	NSR	UFR	USR
Mean	0.000420	0.000420	0.000242	0.000236
SD	0.015757	0.015137	0.005797	0.005892
Skewness	0.181725	0.303064	0.141431	0.121624
Kurtosis	15.98778	17.19066	6.223010	6.992557
Jarque-Bera	10,740.83	12,835.85	666.0141	1,017.980
Probability	0.000000	0.000000	0.000000	0.000000
Observations	1,527	1,527	1,527	1,527

of domestic portfolio. Descriptive statistics explains the nature of the data. The mean for the entire sample period of NFR, NSR, UFR and USR is positive. The mean returns of NFR (0.000420) and NSR (0.000420) were high, followed by the mean returns of UFR (0.000242) and USR (0.000236). The INRUSD future currency pair showed high mean return than the INRUSD spot currency fair. The NFR shows highest volatility with high standard deviation (1.57%), followed by NSR (1.51%). USDINR spot currency pair (USR) showed high volatility with standard deviation of 0.58% compared to USDINR future currency pair (UFR) with standard deviation of 0.57%. This indicates that the mean returns and volatility are high in equity market than in the foreign exchange market. The values of skewness and kurtosis show that the returns are leptokurtic. The coefficients of Jarque-Bera statistics are high and as the *p*-value is significant at 5%, the null hypothesis that returns of the series are normally distributed is rejected.

The descriptive statistics of equity market indices returns which are considered for building international portfolios are shown in Table 4. The mean return of all the equity indices shows positive return, except the UK equity market. The mean return (0.05%) of Russian equity market is the highest. Further, high standard deviation of 2.19%, indicating high volatility is observed in the case of Russian equity market. The figures of skewness indicate that all series are asymmetric in nature and negatively skewed. The kurtosis of all the equity market indices is leptokurtic. The Jarque-Bera statistics is significantly high and hence we reject the null hypothesis of normal distribution at 5% significance level.

Classification of Investors

Tables 5 and 6 provide the standard deviation of the asset series in the portfolio and the ranks assigned for both domestic and international portfolios, respectively. For risk-averse investors, the low standard deviation is given high rank and high standard deviation is given the least rank. For risk-neutral investors, all assets with different standard deviations are given equal ranks. For risk-takers, high standard deviation is given high rank and low standard deviation is given low rank. It is evident from Table 5 for domestic portfolio investments, the risk-averse investors invest 40% in USR, 30% in UFR, 20% in NSR and 10% in NFR. Risk-neutral

Table 4: Descriptive Statistics of International Equity Markets

	IBOV INDEX	INDEXCF INDEX	JALSH INDEX	NIFTY INDEX	NYA INDEX	SHCOMP INDEX	SPX INDEX	UKX INDEX
Mean	0.000277	0.000569	0.000453	0.000438	0.000116	0.000220	8.59E-05	–0.000012
SD	0.018238	0.021903	0.012287	0.015638	0.012642	0.015259	0.012667	0.012182
Skewness	–0.082865	–0.191686	–0.158666	–0.248237	–0.316274	–0.075737	–0.173005	–0.151444
Kurtosis	7.024652	16.05572	6.685087	11.15313	12.57541	7.892378	11.20738	9.477446
Jarque-Bera	2,656.891	27,935.57	2,240.193	10,925.42	15,079.51	3,923.172	11,049.97	6,885.532
Probability	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Observations	3,930	3,930	3,930	3,930	3,930	3,930	3,930	3,930

Table 5: Ranks for Domestic Portfolio Investments				
Variables	SD	Ranks		
		Risk-Averse	Risk-Neutral	Risk-Takers
NFR	0.015757	1 (10%)	1 (25%)	4 (40%)
NSR	0.015137	2 (20%)	1 (25%)	3 (30%)
UFR	0.005797	3 (30%)	1 (25%)	2 (20%)
USR	0.005892	4 (40%)	1 (25%)	1 (10%)

Table 6: Ranks for International Portfolio Investments				
Variables	SD	Ranks		
		Risk-Averse	Risk-Neutral	Risk-Takers
IBOV INDEX	0.018238	2 (6%)	1 (13%)	7 (19%)
INDEXCF INDEX	0.021903	1 (3%)	1 (13%)	8 (22%)
JALSH INDEX	0.012287	7 (19%)	1 (13%)	2 (6%)
NIFTY INDEX	0.015638	3 (8%)	1 (13%)	6 (17%)
NYA INDEX	0.012642	6 (17%)	1 (13%)	3 (8%)
SHCOMP INDEX	0.015259	4 (11%)	1 (13%)	5 (14%)
SPX INDEX	0.012667	5 (14%)	1 (13%)	4 (11%)
UKX INDEX	0.012182	8 (22%)	1 (13%)	1 (3%)
Note: For Tables 5 and 6, figures in brackets indicate the percentage of investment.				

investors invest equally around 25% in each market. Risk-takers invest 40% in NFR, 30% in NSR, 20% in UFR and 10% in USR.

As per Table 6, for the international portfolio investment, the risk-averse investors invest 6% in IBOV, 3% in INDEXCF, 19% in JALSH, 8% in NIFTY, 17% in NYA, 11% in SHCOMP, 14% in SPX and 22% in UKX. The risk-neutral investors invest equally around 13% in all the equity indices. The risk-takers invest 19% in IBOV, 22% in INDEXCF, 6% in JALSH, 17% in NIFTY, 8% in NYA, 14% in SHCOMP, 11% in SPX and 3% in UKX. Based on the ranks, the proportion of investments to be invested by the investors is accordingly calculated by providing weightage of each rank to the total of ranks.

Correlation Analysis

Table 7 shows the covariance and correlation analysis between NFR, NSR, USR and UFR. The correlation analysis is carried out in order to know if there is a relationship between the variables considered for the domestic portfolio.

Hypotheses are tested at 5% significance level. From the results shown in Table 7, H_1 and H_6 are rejected. As such, there exists a high positive relationship (0.99) between NSR and NFR. Similarly a high positive relationship (0.92) is found between USR and UFR.

Table 7: Correlation Analysis of Indian Financial Markets				
Null Hypotheses	**Covariance**	**Correlation**	***t*-Statistic**	**Probability**
H_1: *There is no correlation between NSR and NFR*	0.000237	0.992496	316.9714	0.0000
H_2: *There is no correlation between NSR and USR*	–0.00000009	–0.0028	–0.1109	0.9117
H_3: *There is no correlation between NFR and USR*	–0.00000059	–0.0082	–0.3189	0.7498
H_4: *There is no correlation between NSR and UFR*	–0.00000023	–0.0045	–0.1743	0.8616
H_5: *There is no correlation between NFR and UFR*	–0.00000074	–0.0098	–0.3838	0.7012
H_6: *There is no correlation between USR and UFR*	–0.00000074	0.92179	92.8479	0.0000

Hypotheses H_2, H_3, H_4 and H_5 are not rejected as the *p*-values are not significant. Covariance measures the degree to which the mean values of the variables differ from each other. The covariance between USR and UFR (–0.00000074) is found to be the lowest and NSR and NFR is found to be the highest (0.000237).

Table 8 shows the correlation analysis between the international equity markets. The null hypothesis of all the correlation results is rejected at 5% significance level. Hence we conclude that there exists a relationship between the equity markets considered for international portfolio investment. SPX and NYA show the highest correlation of 0.97 which is very apparent as they belong to the same US economy. NYA (0.57) and SPX (0.52) also show moderate correlation with UK equity indices. As such, it can be said that there exists co-movement of equity markets among the developed economies. Among all the BRICS nations, Russian equity market showed high moderate correlation with the developed equity markets of US and UK. Chinese equity market showed the lowest positive correlation with the developed markets of US and UK. Similar results were found for covariance figures.

Unit Root Testing

Table 9 shows the stationarity results using ADF test of all the variables considered for portfolio construction. The null hypothesis of presence of unit root is rejected and it can be concluded that all the variables are stationary at 1% significance level.

Variance-Covariance VaR Analysis

The results of VaR-CoVaR for domestic portfolio and international portfolio are given in Tables 10 and 11, respectively. The need for using this model is based on the principle of portfolio diversification given by Markowitz (1952). It states that risk on investments can be

Table 8: Correlation Analysis of International Equity Markets

Null Hypotheses	Covariance	Correlation	*t*-Statistic	Probability
There is no correlation between IBOV and INDEXCF	0.00012	0.30087	19.7725	0.0000
There is no correlation between IBOV and JALSH	0.0000769	0.34319	22.8998	0.0000
There is no correlation between INDEXCF and JALSH	0.0001300	0.49668	35.8658	0.0000
There is no correlation between NIFTY and IBOV	0.0000634	0.2225	14.3031	0.0000
There is no correlation between NIFTY and INDEXCF	0.0001000	0.29994	19.7058	0.0000
There is no correlation between NIFTY and JALSH	0.0000686	0.35699	23.9521	0.0000
There is no correlation between NYA and IBOV	0.0001400	0.61453	48.8217	0.0000
There is no correlation between NYA and INDEXCF	0.0000858	0.31002	20.437	0.0000
There is no correlation between NYA and JALSH	0.0000605	0.38962	26.5145	0.0000
There is no correlation between NYA and NIFTY	0.0000459	0.23248	14.981	0.0000
There is no correlation between SHCOMP and IBOV	0.0000300	0.10773	6.79137	0.0000
There is no correlation between SHCOMP and INDEXCF	0.0000411	0.1231	7.77444	0.0000
There is no correlation between SHCOMP and JALSH	0.0000260	0.13848	8.76359	0.0000
There is no correlation between SHCOMP and NIFTY	0.0000422	0.17674	11.2542	0.0000

Table 8 (Cont.)

Null Hypotheses	Covariance	Correlation	*t*-Statistic	Probability
There is no correlation between SHCOMP and NYA	0.0000111	0.05779	3.62801	0.0003
There is no correlation between SPX and IBOV	0.0001400	0.58966	45.7579	0.0000
There is no correlation between SPX and INDEXCF	0.0000721	0.25993	16.8708	0.0000
There is no correlation between SPX and JALSH	0.0000527	0.33853	22.548	0.0000
There is no correlation between SPX and NIFTY	0.0000383	0.19326	12.3453	0.0000
There is no correlation between SPX and NYA	0.0001600	0.97676	285.602	0.0000
There is no correlation between SPX and SHCOMP	0.0000063	0.03239	2.0311	0.0423
There is no correlation between UKX and IBOV	0.0001000	0.45796	32.2868	0.0000
There is no correlation between UKX and INDEXCF	0.0001300	0.47465	33.7979	0.0000
There is no correlation between UKX and JALSH	0.0000875	0.58468	45.1688	0.0000
There is no correlation between UKX and NIFTY	0.0000607	0.31892	21.0895	0.0000
There is no correlation between UKX and NYA	0.0000888	0.57656	44.2266	0.0000
There is no correlation between UKX and SHCOMP	0.0000176	0.09462	5.95713	0.0000
There is no correlation between UKX and SPX	0.0000815	0.52837	39.004	0.0000

Table 9: Unit Root Test Using Augmented Dickey Fuller Test		
Null Hypotheses	**ADF *t*-Statistic**	**Probability**
NFR has a Unit Root	–37.15883	0.0000
NSR has a Unit Root	–36.09899	0.0000
USR has a Unit Root	–37.49917	0.0000
UFR has a Unit Root	–37.21800	0.0000
IBOV has a Unit Root	–64.78381	0.0001
INDEXCF has a Unit Root	–61.45657	0.0001
NIFTY has a Unit Root	–61.88634	0.0001
SHCOMP has a Unit Root	–63.46528	0.0001
JALSH has a Unit Root	–61.46229	0.0001
NYA has a Unit Root	–68.11675	0.0001
SPX has a Unit Root	–69.59641	0.0001
UKX has a Unit Root	–30.34934	0.0000

Table 10: Domestic Portfolio VaR (in %)

Confidence Interval	Risk-Averse Investors		Risk-Taker Investors		Neutral Investors	
	Portfolio VaR	Failure Rate	Portfolio VaR	Failure Rate	Portfolio VaR	Failure Rate
99%	1.39	3.99	2.51	0.59	1.88	1.44
95%	0.97	8.45	1.77	1.83	1.32	4.58
90%	0.75	13.43	1.37	4.13	1.02	7.92

Table 11: International Portfolio VaR (in %)

Confidence Interval	Risk-Averse Investors		Risk-Taker Investors		Neutral Investors	
	Portfolio VaR	Failure Rate	Portfolio VaR	Failure Rate	Portfolio VaR	Failure Rate
99%	2.13	1.20	2.58	1.22	2.24	1.12
95%	1.50	3.59	1.18	3.44	1.54	3.59
90%	1.17	6.74	1.41	6.34	1.22	6.59

reduced by holding portfolio of various assets, compared to investments in individual assets. The most classical problem faced by the investor is to find the best mix of assets in his portfolio. This depends on the investors' attitude towards risk and his perceived trade-off between risk and return of the investor (Alexander, 2008). Markowitz (1959) provided a solution for the portfolio allocation decision for the risk-averse investors. While constructing the portfolios, the correlations between the asset returns must be considered.

The higher the correlation between the asset returns, the higher is the risk of the portfolio. Risk can be reduced to its maximum level provided there is high negative correlation between asset returns. The choice of assets in the portfolios must be based on the maximum positive and negative variations (Xu *et al.*, 2012). The job of market risk manager is to monitor risks frequently and use appropriate method wherein risks can be aggregated accounting for offsetting the positions and correlations between assets and risk factors (Alexander, 2008).

The assets selected for the domestic and international portfolios meet the criteria of portfolio diversification with either negative correlations or low correlations between the assets considered in the portfolio. In this backdrop, a simple variance covariance VaR model is used to calculate the market risk of a hypothetical portfolio. The variance-covariance approach is considered more appropriate for measuring portfolio VaR as it contains information of volatility and correlations of all market prices used in the portfolio (Darryll, 1996). The portfolio market risk is assessed at 1%, 5% and 10% significance levels for both domestic and international portfolios. The market risk is calculated for risk-averse, risk-takers and risk- neutral investors.

It is seen from Table 10 that as the confidence level increases, the VaR also increases for all categories of risk investors. The VaR for risk-averse investors is relatively less compared to risk-takers and risk-neutral investors. At 99% confidence interval, the VaR for risk-averse is 1.39%, for risk-takers it is 2.51% and for risk-neutral it is 1.44%. At 95%, the VaR for risk-averse investors is 0.97%, for risk-takers it is 1.77% and for risk-neutral investors it is 1.32%. At 90%, the VaR for risk-averse investors is 0.75%, for risk-takers it is 1.37% and for risk-neutral investors it is 1.02%. However, the failure rate states that VaR-CoVaR performs well at all confidence intervals for risk-takers because the failure rates are less than the significance level. For risk-neutral investors, the model holds good at 95% and 90% confidence intervals. For risk-averse investors, the model passes the failure test at 90% confidence interval. Domestic portfolio is a combination of equity, currency and their derivative instruments. Risk-takers are ready to assume more risk by investing more in derivative instruments, i.e., 40% in NFR and 20% in UFR. Further, for risk-takers, the hypothetical portfolio can be considered as good, since there is a possibility of offsetting of positions in case of volatility in the market.

The hypothetical international portfolio comprises BRICS equity indices, two US equity indices and UK equity index. Table 11 shows that as the confidence level increases, VaR also increases for all types of combinations. Further, the failure rate results suggest that the model performs well only at 95% and 90% confidence intervals. However, the VaR for risk-neutral investors is 1.54% which is high relatively. For risk-averse investors, the VaR is 1.50% and for risk-takers the expected loss is 1.18% which is less. But at 90% confidence interval, the VaR for risk-averse investors is 1.17%, for risk-takers it is 1.41% and for risk-neutral investors it is 1.22%.

Conclusion

Given the fact that emerging economies are growing at a faster pace than the developed markets in terms of size and numbers, investors are not confining only to the domestic investment, rather they are venturing into international investments due to growing investment opportunities. Quantifying of international investment prospects is very essential. There was a time when investors looked at the developed markets for international investments. Over the past decade, the emerging economies have shown profound growth in their macroeconomic fundamentals with improved monetary, fiscal and government policies. This has made the investors look at the emerging markets as an important component in their global equity portfolio allocation. Given the projected prospects of BRICS, the study aimed at calculating international market risk by building a hypothetical equity portfolio of BRICS along with US and UK equity indices. The question was whether the portfolio could offer better investment opportunity by minimizing the risk of loss. VaR for the hypothetical portfolio is estimated using variance-covariance model. Variance-covariance VaR model is considered to be the simplest of all for calculating the portfolio VaR. The method takes into consideration the correlations, variances and volatility among the equity indices for measuring the VaR.

Even in the case of portfolio VaR it is observed that as the confidence level increases, VaR also increases. VaR estimates at 95% and 90% confidence intervals give accurate results as the failure rates are less than the expected loss percentage. The portfolio VaR estimates at both 95% and 90% confidence intervals imply that the expected loss for risk-averse investors is less than the risk-neutral investors and risk-takers for both domestic and international portfolio investments. ⅄

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